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Frame-like gauge invariant Lagrangian formulation of massive fermionic higher spin fields in AdS_3 space

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ABSTRACT

We construct the frame-like gauge-invariant Lagrangian formulation for massive fermionic arbitrary spin fields in three-dimensional *AdS* space. The Lagrangian and complete set of gauge transformations are obtained. We also develop the formalism of gauge-invariant curvatures for the massive theory under consideration and show how the Lagrangian is formulated in their terms. The massive spin-5/2 field is discussed as an example.

1. Introduction

Despite the significant progress in the theory of massless higher-spin fields in various dimensions (see reviews [1–3], see also [4] for details), the construction of full higher spin field theory is still far from being complete. In particular it remains unclear how the non-linear theory of massive higher spins should look like. Investigations of massive higher spin interactions are very important in-particular for understanding the relation between higher spins and (super)string theories that is assumed to realize a kind of spontaneous symmetry breaking mechanism. Thus a natural framework for such investigations is a gauge invariant formulation for massive higher spin fields (similar to the one appearing in string field theory) that becomes possible due to introduction of appropriate number of Stueckelberg fields. Currently gauge invariant description for the free massive higher-spin fields is rather well studied, moreover various authors have developed different ways of such description [5–9].

At the same time studying the massive higher spin interactions in arbitrary dimensions appears to be even more complicated that the ones for the massless fields. Therefore it would be instructive to investigate a structure of the massive higher spin theories coupled to external fields or among themselves in three-dimensional space where situation becomes simpler than in a space of arbitrary dimension. As a result we can get a nice playground to gain useful experience and possibilities for generalizations. In particular it turns out that in $d = 3$ there exist examples of interacting models with finite number of higher spin fields [10–13]. Moreover the specific properties of three dimensional space allow us to construct the more exotic higher spin models [14]. Therefore we can expect that the massive higher-spin theory also becomes easier in $d = 3$.

First Lagrangian formulation for massive higher-spin fields in $d = 3$ was considered in [15] however in gauge invariant form it has been developed for bosonic fields only [16–18]. In this work we fill this gap and give gauge invariant formulation for massive fermionic higher spins in three-dimensional AdS_3 space. In this we think that the most convenient formalism is the frame-like one [19–21], which in particular allows one to work with an explicitly invariant objects.

The paper is organized as follows. In Section 2 we collect the basic information about frame-like formulation of free massless higher spins in AdS_3 focusing on the fermionic fields only. In Section 3 we give the frame-like gauge-invariant formulation for free massive fermionic higher spins in AdS_3. Here at first we discuss a field content which we need to have gauge invariant description of massive fields. Then in terms of these fields we derive the Lagrangian and corresponding gauge transformations. We also construct the full set of gauge invariant linearized curvatures and show how the free Lagrangian can be rewritten in their terms. At the end of Section 3 we consider the concrete example of massive spin-5/2 field. In conclusion we summarize the main points.

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2. Massless fermionic fields

In this section we briefly review a frame-like formulation of massless higher spin fermionic fields in AdS_3 space. The Lagrangian, gauge transformation and gauge invariant curvatures for free fields are given.

Frame-like formulation of massless higher-spin fields can be treated as generalization of the frame formulation of gravity in terms of vielbein field $h_\mu{}^a$ and Lorentz connection $\omega_\mu{}^{a,b}$. Such generalization was successfully developed both for the bosonic fields and for the fermionic ones [19–21]. We will focus here only on half-integer spins, the details for integer spins can be found in [19,20].

It is known [19] that in four dimensions the field with half-integer spin $s > 3/2$ in the frame-like formulation is described by a total bunch of spin-tensor one-forms (here we omit spinor index)

$$\Psi^{a_1\dots a_{s-3/2},b_1\dots b_k} = dx^\mu \Psi_\mu{}^{a_1\dots a_{s-3/2},b_1\dots b_k}, \quad 0 \le k \le s - 3/2$$

These fields on Latin indices form an irreducible representations of the Lorentz group, i.e. satisfy the symmetry of two-row Young tableaux and are γ-traceless. Note that for $k = 0$ we have the generalized vielbein field $\Psi^{a_1\dots a_{s-3/2}}$ while for other values of k we have so-called extra fields. An important property of $d = 3$ case is the absence of these extra fields that greatly simplifies the calculations in construction of consistent higher spin field theories. Moreover the description can be simplified even more by the use of multispinor frame-like formalism where all fields are still one-forms but with all local indices replaced by the spinor ones [11]. The main object then is completely symmetric multispinor one-form $\Psi^{\alpha(n)}$ (see Appendix A for notations and conventions) with $n = 2(s - 1)$ (where s is half-integer and n is odd) which is equivalent to the completely symmetric spin-tensor $\Psi^{a_1\dots a_{s-3/2}}$ satisfying γ-tracelessness condition $(\gamma\Psi)^{a_1\dots a_{s-5/2}} = 0$.

Free Lagrangian being three-form in three dimensional AdS_3 space looks as follows [1,11]

$$\mathcal{L}_0 = i\kappa_n\left[\Psi_{\alpha(n)} D\Psi^{\alpha(n)} + \frac{n\lambda}{2}\Psi_{\alpha(n-1)\beta} e^\beta{}_\gamma \Psi^{\alpha(n-1)\gamma}\right] \tag{1}$$

here $D = dx^\mu D_\mu$ is AdS_3 covariant derivative, one-form $e^{\alpha(2)}$ is AdS_3 background vielbein (see Appendix A for details) and $\kappa_n = (-1)^{\frac{n+1}{2}}$. The Lagrangian (1) is invariant under the gauge transformations

$$\delta_0\Psi^{\alpha(n)} = D\xi^{\alpha(n)} + \frac{\lambda}{2} e^\alpha{}_\beta \xi^{\alpha(n-1)\beta}$$

with zero-form gauge parameter $\xi^{\alpha(n)}$. For this field $\Psi^{\alpha(n)}$ there exists the two-form gauge invariant object (curvature)

$$\mathcal{R}^{\alpha(n)} = D\Psi^{\alpha(n)} + \frac{\lambda}{2} e^\alpha{}_\beta \Psi^{\alpha(n-1)\beta}$$

which satisfies the Bianchi identity

$$D\mathcal{R}^{\alpha(n)} = -\frac{\lambda}{2} e^\alpha{}_\beta \mathcal{R}^{\alpha(n-1)\beta}$$

Using this curvature the free Lagrangian can be rewritten as follows

$$\mathcal{L}_0 = i\kappa_n \Psi_{\alpha(n)} \mathcal{R}^{\alpha(n)}$$

In order to verify gauge invariance of the Lagrangian in such form we should use the Bianchi identity. It is easy to see that the curvature just gives the equations of motion for the Lagrangian (1). Note that the possibility to work in terms of the curvatures is a peculiarity of the frame-like formalism.

3. Massive fermionic fields

In this section we develop the frame-like gauge invariant description for massive arbitrary half-integer spin $s \ge 3/2$ in AdS_3 space. To provide the gauge invariance we introduce the auxiliary Stueckelberg fields and construct the Lagrangian in their terms. Then we generalize the formalism of gauge-invariant curvatures for massive fields. General formulation is illustrated on the example of massive spin-5/2 theory.

3.1. Lagrangian formulation

Gauge invariant Lagrangian formulation of massive fields is based on introduction of the auxiliary Stueckelberg fields. We follow the procedure proposed in [6] to use the minimal number of such fields. In the case under consideration the full set of field variables includes the following one-forms $\Psi^{\alpha(n)}$, $n = 1, 3, \dots, 2(s - 1)$ (each one with its own gauge transformations) and zero-form ϕ^α. We will look for the free Lagrangian for massive field as the sum of kinetic terms for all these fields as well as the most general mass-like terms gluing them together:

$$\begin{aligned}\mathcal{L}_0 = {} & i\sum_{n=1}^{2(s-1)} \frac{\kappa_n}{2}\Psi_{\alpha(n)} D\Psi^{\alpha(n)} + \frac{i}{2}\phi_\alpha E_2{}^\alpha{}_\beta D\phi^\beta \\ & + i\sum_{n=3}^{2(s-1)} \kappa_n a_n \Psi_{\alpha(n)} e^{\alpha\alpha}\Psi^{\alpha(n-2)} + ia_0\Psi_{\alpha(1)} E_2{}^\alpha{}_\beta \phi^\beta \\ & + i\sum_{n=1}^{2(s-1)} \frac{\kappa_n b_n}{2}\Psi_{\alpha(n)} e^\alpha{}_\beta \Psi^{\alpha(n-1)\beta} + i\frac{b_0}{2}\phi_\alpha E_3\phi^\alpha\end{aligned} \tag{2}$$

here $\kappa_n = (-1)^{\frac{n+1}{2}}$ and a_n, b_n are free parameters to be determined. The most general form of the corresponding gauge transformations looks like

$$\begin{aligned}\delta_0\Psi^{\alpha(n)} &= D\xi^{\alpha(n)} + \alpha_n e^\alpha{}_\beta \xi^{\alpha(n-1)\beta} \\ &\quad + \beta_n e^{\alpha\alpha}\xi^{\alpha(n-2)} + \gamma_n e_{\beta\beta}\xi^{\alpha(n)\beta\beta} \\ \delta_0\phi^\alpha &= \alpha_0\xi^\alpha\end{aligned} \tag{3}$$

where α_n, β_n, γ_n are also free parameters. Our aim is to find the restrictions on the parameters in the Lagrangian and gauge transformations providing the gauge invariance of the Lagrangian (2) under the transformations (3).

First of all we consider variations of the Lagrangian that are of the first order in derivatives. This allows us to express the parameters a_n, b_n in the Lagrangian through parameters in the gauge transformations

$$a_n = \frac{n(n-1)}{2}\beta_n, \qquad b_n = n\alpha_n, \qquad a_0 = \alpha_0 \tag{4}$$

and also imposes one restriction on the parameters of the gauge transformations

$$\beta_n = \frac{2}{n(n-1)}\gamma_{n-2} \tag{5}$$

Note that this condition is valid because β_n is defined for $3 \le n \le 2(s - 1)$ and γ_n for $1 \le n \le 2(s - 2)$. The remaining free parameters α_n, γ_n are fixed from the invariance conditions under the transformations (3) for the variations without derivatives. Direct calculations yield the following equations

$$2(n+2)a_n\alpha_n - 2b_{n-2}\gamma_{n-2} = 0 \quad n \ge 3$$

$$2(n-2)a_n\alpha_{n-2} - (n-1)(n+2)b_n\beta_n = 0 \quad n \ge 3$$

$$-n\lambda^2 - 4a_n\gamma_{n-2} + 4b_n\alpha_n + 2n(n+3)a_{n+2}\beta_{n+2} = 0 \quad n \geq 3$$
$$-\lambda^2 + 4b_1\alpha_1 + 8a_3\beta_3 + a_0\alpha_0 = 0 \quad n = 1$$
$$3a_0\alpha_1 + b_0\alpha_0 = 0 \tag{6}$$

Last equation allows us to express the only remaining free parameter in the Lagrangian b_0

$$b_0 = -3\alpha_1$$

Taking into account the relations (4), (5) we see that first two equations in (6) are identical and lead to a simple recurrent relation

$$(n-2)\alpha_{n-2} - (n+2)\alpha_n = 0$$

Denoting the maximal value of $n = 2(s-1) = \hat{s}$ and expressing everything through $\alpha_{\hat{s}}$ we get the general solution for α_n in the form

$$\alpha_n = \frac{\hat{s}(\hat{s}+2)}{n(n+2)}\alpha_{\hat{s}} \tag{7}$$

Now let us consider the third equation (6) containing two parameters

$$-n\lambda^2 + 4n\alpha_n{}^2 - 4\gamma_{n-2}{}^2 + \frac{4n(n+3)}{(n+1)(n+2)}\gamma_n{}^2 = 0 \tag{8}$$

Here we have used the relations (4), (5). First, note that the γ_n is absent for $n = \hat{s}$. Therefore one can express $\alpha_{\hat{s}}$ through $\gamma_{\hat{s}-2}$ (as we will see later the parameter $\gamma_{\hat{s}-2}$ will remain as the only free one and will play the role of a mass parameter)

$$\alpha_{\hat{s}}{}^2 = \frac{1}{\hat{s}}\gamma_{\hat{s}-2}{}^2 + \frac{1}{4}\lambda^2 \tag{9}$$

Taking into account (7), we see that the relation (8) is a recurrent equation for parameters γ_n. General solution looks like

$$\gamma_{n-2}{}^2 = \frac{(\hat{s}-n+2)(\hat{s}+n+2)}{4n(n+1)}\left[m^2 + \frac{1}{4}(\hat{s}-n)(\hat{s}+n)\lambda^2\right] \tag{10}$$

where we introduced mass parameter $m^2 = \hat{s}\gamma_{s-2}{}^2$. Finally from the fourth equation of system (6) it follows that

$$\alpha_0{}^2 = \frac{(\hat{s}+1)(\hat{s}+3)}{2}\left[m^2 + \frac{1}{4}(\hat{s}-1)(\hat{s}+1)\lambda^2\right] \tag{11}$$

Let us also introduce convenient combination

$$M^2 = m^2 + \frac{1}{4}\hat{s}^2\lambda^2 \tag{12}$$

so that now

$$\alpha_n = \frac{(\hat{s}+2)}{n(n+2)}M \tag{13}$$

Thus all parameters are found and are defined by (4), (5), (10)–(13) with m^2 as the only free one. Then the final expression for the Lagrangian (2) takes the form:

$$\begin{aligned}\mathcal{L}_0 = {} & i\sum_{n=1}^{2(s-1)}\frac{\kappa_n}{2}\Psi_{\alpha(n)}D\Psi^{\alpha(n)} + \frac{i}{2}\phi_\alpha E_2{}^\alpha{}_\beta D\phi^\beta \\ & + i\sum_{n=3}^{2(s-1)}\kappa_n m_n\Psi_{\alpha(n)}e^{\alpha\alpha}\Psi^{\alpha(n-2)} + im_0\Psi_{\alpha(1)}E_2{}^\alpha{}_\beta\phi^\beta \\ & + i\sum_{n=1}^{2(s-1)}\frac{(\hat{s}+2)\kappa_n M}{2(n+2)}\Psi_{\alpha(n)}e^\alpha{}_\beta\Psi^{\alpha(n-1)\beta} \\ & - i\frac{(\hat{s}+2)M}{2}\phi_\alpha E_3\phi^\alpha\end{aligned} \tag{14}$$

where we denote $\gamma_{n-2} = m_n$ and $\alpha_0 = m_0$ while the gauge transformations (3) look as follows

$$\begin{aligned}\delta_0\Psi^{\alpha(n)} = {} & D\xi^{\alpha(n)} + \frac{(\hat{s}+2)M}{2n(n+2)}e^\alpha{}_\beta\xi^{\alpha(n-1)\beta} \\ & + \frac{2}{n(n-1)}m_n e^{\alpha\alpha}\xi^{\alpha(n-2)} + m_{n+2}e_{\beta\beta}\xi^{\alpha(n)\beta\beta} \\ \delta_0\phi^\alpha = {} & m_0\xi^\alpha\end{aligned} \tag{15}$$

Note that such gauge invariant description works not only in *AdS* (and Minkowski) space but in *dS* space as well provided $m^2 > \frac{1}{4}\hat{s}^2\Lambda$, $\Lambda = -\lambda^2$ so that massless limit is possible in the Minkowski and *AdS* spaces only. Inside the unitary forbidden region there exists a number of partially massless cases [22–24,5,6]. Namely each time when one of the parameters m_n becomes zero the whole system decomposes into two disconnected ones. One of them with the fields $\Psi^{\alpha(2s-1)}\dots\Psi^{\alpha(n)}$ describes partially massless field while the remaining fields describe massive field with spin $\frac{n}{2} - 1$.

3.2. Formulation in terms of curvatures

In this subsection we develop the formalism of gauge invariant curvatures for massive higher spin fermionic field in AdS_3 and rewrite the Lagrangian, found in previous subsection, in terms of these curvatures. The gauge invariant curvatures have been introduced before to construct the Lagrangian formulation for massless higher spin fields in frame-like approach. We will show that the objects with analogous properties can also be constructed for massive higher spin fields. Since the gauge-invariant description of massive fields uses the auxiliary Stueckelberg fields, it is natural to construct the corresponding curvatures for them as well.

First of all we introduce the additional auxiliary zero-forms $C^{\alpha(n)}$, $n = 3, \dots, 2(s-1)$[1] which transform under the gauge transformations as follows

$$\delta_0 C^{\alpha(n)} = \eta_n\xi^{\alpha(n)} \tag{16}$$

where the parameters η_n are yet to be fixed. We combine these fields with our spinor field denoting $\phi^\alpha = C^\alpha$ and similarly for parameter of gauge transformations $\eta_1 = \alpha_0$. The most general ansatz for the full set of curvatures looks like

$$\begin{aligned}\mathcal{R}^{\alpha(n)} = {} & D\Psi^{\alpha(n)} + \alpha_n e^\alpha{}_\beta\Psi^{\alpha(n-1)\beta} + \beta_n e^{\alpha\alpha}\Psi^{\alpha(n-2)} \\ & + \gamma_n e_{\beta\beta}\Psi^{\alpha(n)\beta\beta} + f_n E_2{}^\alpha{}_\beta C^{\alpha(n-1)\beta} \\ \mathcal{C}^{\alpha(n)} = {} & DC^{\alpha(n)} + l_n\Psi^{\alpha(n)} + k_n e^\alpha{}_\beta C^{\alpha(n-1)\beta} \\ & + q_n e^{\alpha\alpha}C^{\alpha(n-2)} + p_n e_{\beta\beta}C^{\alpha(n)\beta\beta}\end{aligned}$$

where α_n, β_n, γ_n are the same as in previous subsection and guarantee that curvature $\mathcal{R}$ is invariant under the part of gauge transformation containing derivatives. Other parameters are arbitrary and will be fixed by the gauge invariance under the transformations (3), (16). Here $\mathcal{C}^{\alpha(n)}$ are the curvatures corresponding to the zero-forms $C^{\alpha(n)}$.

Parameters l_n, k_n, q_n, p_n are found from gauge invariance for curvatures $\mathcal{C}$. We have

$$\begin{aligned}\delta\mathcal{C}^{\alpha(n)} = {} & D\left(\eta_n\xi^{\alpha(n)}\right) + l_n\left(D\xi^{\alpha(n)} + \alpha_n e^\alpha{}_\beta\xi^{\alpha(n-1)\beta}\right. \\ & \left. + \beta_n e^{\alpha\alpha}\xi^{\alpha(n-2)} + \gamma_n e_{\beta\beta}\xi^{\alpha(n)\beta\beta}\right) \\ & + k_n e^\alpha{}_\beta\left(\eta_n\xi^{\alpha(n-1)\beta}\right) + q_n e^{\alpha\alpha}\left(\eta_{n-2}\xi^{\alpha(n-2)}\right) \\ & + p_n e_{\beta\beta}\left(\eta_{n+2}\xi^{\alpha(n)\beta\beta}\right)\end{aligned}$$

[1] These fields are just the first representatives of infinite number of zero-forms present in a full unfolded formulation. They do not enter the free Lagrangian but we need them to construct gauge invariant objects for physical fields.

It leads to the relations

$$l_n + \eta_n = 0 \qquad l_n\beta_n + q_n\eta_{n-2} = 0$$
$$l_n\alpha_n + k_n\eta_n = 0 \qquad l_n\gamma_n + p_n\eta_{n+2} = 0$$

Using the arbitrariness in definition of $C^{\alpha(n)}$ related with multiplication by constant, $C^{\alpha(n)} \to c_n C^{\alpha(n)}$ (c_n are some numerical coefficients) we can set $p_n = \gamma_n$. Then the general solution of the relations above looks like

$$l_n = -\eta_n, \qquad k_n = \alpha_n, \qquad q_n = \beta_n, \qquad \eta_n = \eta_{n+2} \tag{17}$$

The last condition is the recurrent relation. To solve it we use as initial data $\eta_1 = \alpha_0$ and then obtain $\eta_n = \eta_1 = \alpha_0$.

To find the last parameters f_n we consider the invariance of the curvature $\mathcal{R}^{\alpha(n)}$ under the gauge transformations without derivatives. Invariance under the transformations containing the derivatives has been established earlier. Corresponding variations can be written in the form

$$\begin{aligned}
\delta\mathcal{R}^{\alpha(n)} = {} & -\lambda^2 E_2{}^\alpha{}_\beta \xi^{\alpha(n-1)\beta} \\
& + \alpha_n\big(4\alpha_n E_2{}^\alpha{}_\gamma \xi^{\alpha(n-1)\gamma} + 2(n-2)\beta_n E_2{}^{\alpha\alpha}\xi^{\alpha(n-2)} \\
& + 8\beta_n E_2{}^{\alpha\alpha}\xi^{\alpha(n-1)} - 2n\gamma_n E_{2\gamma\gamma}\xi^{\alpha(n)\gamma\gamma}\big) \\
& + \beta_n\big(-2(n-2)\alpha_{n-2}E_2{}^{\alpha\alpha}\xi^{\alpha(n-2)} \\
& - 2(n-1)\gamma_{n-2}E_2{}^\alpha{}_\beta\xi^{\alpha(n-1)\beta}\big) \\
& + \gamma_n\big(2n\alpha_{n+2}E_{2\beta\beta}\xi^{\alpha(n)\beta\beta} \\
& + 2(n-1)\beta_{n+2}E_{2\beta}{}^\alpha\xi^{\alpha(n-1)\beta} + 8\beta_{n+2}E_{2\beta}{}^\alpha\xi^{\alpha(n-1)\beta}\big) \\
& + f_n\eta_n E_2{}^\alpha{}_\beta\xi^{\alpha(n-1)\beta}
\end{aligned}$$

Invariance condition leads to

$$-\lambda^2 + 4\alpha_n{}^2 - 2(n-1)\beta_n\gamma_{n-2} + 2(n+3)\gamma_n\beta_{n+2} + f_n\eta_n = 0$$
$$2(n+2)\alpha_n\beta_n - 2(n-2)\beta_n\alpha_{n-2} = 0$$

Comparing with relations (6), we conclude that

$$f_1 = \alpha_0, \qquad f_n = 0, \quad n \geq 3$$

As a result we have the final expressions for the curvatures

$$\begin{aligned}
\mathcal{R}^{\alpha(n)} &= D\Psi^{\alpha(n)} + \alpha_n e^\alpha{}_\beta \Psi^{\alpha(n-1)\beta} \\
&\quad + \beta_n e^{\alpha\alpha}\Psi^{\alpha(n-2)} + \gamma_n e_{\beta\beta}\Psi^{\alpha(n)\beta\beta} \\
\mathcal{R}^\alpha &= D\Psi^\alpha + \alpha_1 e^\alpha{}_\beta\Psi^\beta + \gamma_1 e_{\beta\beta}\Psi^{\alpha\beta\beta} + \alpha_0 E_2{}^\alpha{}_\beta\phi^\beta \\
\mathcal{C}^\alpha &= D\phi^\alpha - \alpha_0\Psi^\alpha + \alpha_1 e^\alpha{}_\beta\phi^\beta + \gamma_1 e_{\beta\beta}C^{\alpha\beta\beta} \\
\mathcal{C}^{\alpha(n)} &= DC^{\alpha(n)} - \alpha_0\Psi^{\alpha(n)} + \alpha_n e^\alpha{}_\beta C^{\alpha(n-1)\beta} \\
&\quad + \beta_n e^{\alpha\alpha}C^{\alpha(n-2)} + \gamma_n e_{\beta\beta}C^{\alpha(n)\beta\beta}
\end{aligned} \tag{18}$$

Here the curvatures corresponding to $n = 1$ are written separately.

Now let us rewrite the Lagrangian (2) in terms of curvatures. The most general expression for it has the form

$$\mathcal{L}_0 = i\sum_{n=1}^{2(s-1)} A_n\Psi_{\alpha(n)}\mathcal{R}^{\alpha(n)} + i\sum_{n=1}^{2(s-1)} B_n C_{\alpha(n-1)\beta}E_2{}^\beta{}_\gamma \mathcal{C}^{\alpha(n-1)\gamma}$$

where A_n and B_n are the arbitrary coefficients and are fixed by requirement to reproduce initial Lagrangian (2). Since the fields $C^{\alpha(n)}$ for $n \geq 3$ do not enter in (2) we can immediately put $B_n = 0$, $n \geq 3$. Redefining $C^\alpha = \phi^\alpha$, $B_1 = A_0$ ones get

$$\mathcal{L}_0 = i\sum_{n=1}^{2(s-1)} A_n\Psi_{\alpha(n)}\mathcal{R}^{\alpha(n)} + iA_0\phi_\alpha E_2{}^\alpha{}_\beta\mathcal{C}^\beta \tag{19}$$

Comparing (19) with Lagrangian (2) we obtain

$$A_n = \frac{\kappa_n}{2}, \qquad A_0 = \frac{1}{2}$$

As a result we rewrite the Lagrangian (2) in terms of gauge invariant curvatures (19).

3.3. Massive spin 5/2 example

Massive field with spin $s = 5/2$ is described by a set of fields $\Psi^{\alpha(3)}$, Ψ^α, ϕ^α, $C^{\alpha(3)}$, where the second and third fields are Stueckelberg ones while the latter is an auxiliary field and enter in the expressions for curvatures only. Then in accordance with (14) the Lagrangian looks like

$$\begin{aligned}
\mathcal{L}_0 = {} & \frac{i}{2}\Psi_{\alpha(3)}D\Psi^{\alpha(3)} - \frac{i}{2}\Psi_\alpha D\Psi^\alpha + \frac{i}{2}\phi_\alpha E_2{}^\alpha{}_\beta D\phi^\beta \\
& + im_3\Psi_{\alpha(3)}e^{\alpha\alpha}\Psi^\alpha + im_0\Psi_{\alpha(1)}E_2{}^\alpha{}_\beta\phi^\beta \\
& + \frac{iM}{2}\Psi_{\alpha(3)}e^\alpha{}_\beta\Psi^{\alpha(2)\beta} - \frac{5iM}{6}\Psi_\alpha e^\alpha{}_\beta\Psi^\beta - \frac{5iM}{2}\phi_\alpha E_3\phi^\alpha
\end{aligned}$$

where

$$M^2 = m^2 + \frac{9}{4}\lambda^2, \qquad m^2 = 3m_3{}^2, \qquad m_0{}^2 = 12\big(m^2 + 2\lambda^2\big)$$

According to (18) the expressions for the linearized gauge-invariant curvatures have the form

$$\begin{aligned}
\mathcal{R}^{\alpha(3)} &= D\Psi^{\alpha(3)} + \frac{M}{3}e^\alpha{}_\beta\Psi^{\alpha(2)\beta} + \frac{m_3}{3}e^{\alpha\alpha}\Psi^\alpha \\
\mathcal{R}^\alpha &= D\Psi^\alpha + \frac{5M}{3}e^\alpha{}_\beta\Psi^\beta + m_3 e_{\beta\beta}\Psi^{\alpha\beta\beta} + m_0 E_2{}^\alpha{}_\beta\phi^\beta \\
\mathcal{C}^\alpha &= D\phi^\alpha - m_0\Psi^\alpha + \frac{5M}{3}e^\alpha{}_\beta\phi^\beta + m_3 e_{\beta\beta}C^{\alpha\beta\beta} \\
\mathcal{C}^{\alpha(3)} &= DC^{\alpha(3)} - m_0\Psi^{\alpha(3)} + \frac{M}{3}e^\alpha{}_\beta C^{\alpha(2)\beta} + \frac{m_3}{3}e^{\alpha\alpha}\phi^\alpha
\end{aligned}$$

The set of gauge transformations is written as follows

$$\begin{aligned}
\delta_0\Psi^{\alpha(3)} &= D\xi^{\alpha(3)} + \frac{M}{3}e^\alpha{}_\beta\xi^{\alpha(2)\beta} + \frac{m_3}{3}e^{\alpha\alpha}\xi^\alpha \\
\delta_0\Psi^\alpha &= D\xi^\alpha + \frac{5M}{3}e^\alpha{}_\beta\xi^\beta + m_3 e_{\beta\beta}\xi^{\alpha\beta\beta} \\
\delta_0\phi^\alpha &= m_0\xi^\alpha \\
\delta_0 C^{\alpha(3)} &= m_0\xi^{\alpha(3)}
\end{aligned}$$

Accordingly to (19) the Lagrangian in term of curvatures can be written in the form

$$\mathcal{L}_0 = \frac{i}{2}\Psi_{\alpha(3)}\mathcal{R}^{\alpha(3)} - \frac{i}{2}\Psi_\alpha\mathcal{R}^\alpha + \frac{i}{2}\phi_\alpha E_2{}^\alpha{}_\beta\mathcal{C}^\beta$$

4. Conclusion

In this paper we have formulated a gauge-invariant Lagrangian description for massive fermionic higher spins in three-dimensional AdS_3 space. Using suitable set of Stueckelberg fields we have derived the gauge-invariant Lagrangian and obtained a full set of corresponding gauge transformations. We have also constructed a complete set of linearized gauge-invariant curvatures on AdS_3 and found that the gauge invariance required to introduce an additional set of auxiliary fields which do not enter the free Lagrangian. Massive spin-5/2 example is considered in details. We hope that our results provide a ground for the development of frame-like gauge-invariant formalism for interacting massive half-integer spins in three dimensional space.

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Appendix A

In three dimensional space it is convenient to use two-component spinor formalism and, in contrast to the $d = 4$, only one type of spinor indices is used. For instance AdS_3 background vielbein is described by one-form $e^{\alpha(2)}$ and massless arbitrary half-integer spin s is described by one-form $\Psi^{\alpha(n)}$, $n = 2(s-1)$ is odd (here $\alpha = 1, 2$ is spinor index) where the argument of index means the number of totally symmetrized indices. Further we present the list of our notations and conventions for two-component spinor formalism and differential form language (sign $\wedge$ of wedge product in the text of the paper is everywhere omitted):

- For spinor indices labeled by one letter and standing on the same level we use agreement of total symmetrization without normalization factor for example

$$e^\alpha{}_\beta \wedge \Psi^{\alpha(n-1)\beta} = e^{(\alpha_1}{}_\beta \wedge \Psi^{\alpha_2\dots\alpha_n)\beta} = e^{\alpha_1}{}_\beta \wedge \Psi^{\alpha_2\dots\alpha_n\beta} + (n-1) \text{ sym. terms}$$

- For anti-symmetric matrices $\varepsilon^{\alpha\beta}$, $\varepsilon_{\alpha\beta}$ we use the following basic relation and rule for lowering and raising spinor indices

$$\varepsilon^{\alpha\gamma}\varepsilon_{\gamma\beta} = -\delta^\alpha{}_\beta, \qquad \varepsilon^{\alpha\beta}A_\beta = A^\alpha, \qquad \varepsilon_{\alpha\beta}A^\beta = -A_\alpha$$

- Basis elements of $1, 2, 3$-form spaces are respectively $e^{\alpha(2)}$, $E_2{}^{\alpha(2)}$, E_3 where the last two are defined as double and triple wedge product of $e^{\alpha(2)}$

$$e^{\alpha\alpha} \wedge e^{\beta\beta} = \varepsilon^{\alpha\beta}E_2{}^{\alpha\beta}$$

$$E_2{}^{\alpha\alpha} \wedge e^{\beta\beta} = \varepsilon^{\alpha\beta}\varepsilon^{\alpha\beta}E_3$$

  Let us write useful relations for the basis elements

$$E_2{}^\alpha{}_\gamma \wedge e^{\gamma\beta} = 3\varepsilon^{\alpha\beta}E_3, \qquad e^\alpha{}_\gamma \wedge e^{\gamma\beta} = 4E_2{}^{\alpha\beta}$$

$$e^\alpha{}_\beta \wedge e^{\alpha\alpha} = 2\varepsilon_\beta{}^\alpha E_2{}^{\alpha\alpha}, \qquad E_2{}^\alpha{}_\beta \wedge e^{\alpha\alpha} = E_2{}^{\alpha\alpha} \wedge e^\alpha{}_\beta = 0$$

- For AdS_3 covariant derivative we use convention

$$D \wedge D\xi^\alpha = -\lambda^2 E_2{}^\alpha{}_\beta \xi^\beta$$

References

[1] M.A. Vasiliev, Higher-spin gauge theories in four, three and two dimensions, Int. J. Mod. Phys. D 05 (1996) 763.
[2] X. Bekaert, S. Cnockaert, C. Iazeolla, M.A. Vasiliev, Nonlinear higher spin theories in various dimensions, arXiv:hep-th/0503128.
[3] M.A. Vasiliev, Higher spin gauge theories in various dimensions, Fortschr. Phys. 52 (2005) 702, arXiv:hep-th/0401177.
[4] V.E. Didenko, E.D. Skvortsov, Elements of Vasiliev theory, arXiv:1401.2975.
[5] R.R. Metsaev, Gauge invariant formulation of massive totally symmetric fermionic fields in (A)dS space, Phys. Lett. B 643 (2006) 205, arXiv:hep-th/0609029.
[6] Yu.M. Zinoviev, Frame-like gauge invariant formulation for massive high spin particles, Nucl. Phys. B 808 (2009) 185, arXiv:0808.1778.
[7] D.S. Ponomarev, M.A. Vasiliev, Frame-like action and unfolded formulation for massive higher-spin fields, Nucl. Phys. B 839 (2010) 466, arXiv:1001.0062.
[8] I.L. Buchbinder, V.A. Krykhtin, Gauge invariant Lagrangian construction for massive bosonic higher spin fields in D dimensions, Nucl. Phys. B 727 (2005) 537, arXiv:hep-th/0505092.
[9] I.L. Buchbinder, V.A. Krykhtin, L.L. Ryskina, H. Takata, Gauge invariant Lagrangian construction for massive higher spin fermionic fields, Phys. Lett. B 641 (2006) 386, arXiv:hep-th/0603212.
[10] C. Aragone, S. Deser, Hypersymmetry in $D = 3$ of coupled gravity-massless spin-5/2 system, Class. Quantum Gravity 1 (1984) L9.
[11] M.P. Blencowe, A consistent interacting massless higher-spin field theory in $D = 2 + 1$, Class. Quantum Gravity 6 (1989) 443.
[12] A. Campoleoni, S. Fredenhagen, S. Pfenninger, S. Theisen, Asymptotic symmetries of three-dimensional gravity coupled to higher-spin fields, J. High Energy Phys. 11 (2010) 007, arXiv:1008.4744.
[13] B. Chen, J. Long, Y. Wang, Black holes in truncated higher spin AdS_3 gravity, J. High Energy Phys. 12 (2012) 052, arXiv:1209.6185.
[14] E.A. Bergshoeff, O. Hohm, P.K. Townsend, On higher derivatives in 3d gravities and higher spin gauge theories, Ann. Phys. 325 (2010) 1118, arXiv:0911.3061.
[15] I. Tyutin, M. Vasiliev, Lagrangian formulation of irreducible massive fields of arbitrary spin in $2 + 1$ dimensions, Theor. Math. Phys. 113 (1997) 1244, arXiv:hep-th/9704132.
[16] Yu.M. Zinoviev, On massive gravity and bigravity in three dimensions, Class. Quantum Gravity 30 (2013) 055005, arXiv:1205.6892.
[17] I.L. Buchbinder, T.V. Snegirev, Yu.M. Zinoviev, Gauge invariant Lagrangian formulation of massive higher spin fields in AdS_3 space, Phys. Lett. B 716 (2012) 243, arXiv:1207.1215.
[18] I.L. Buchbinder, T.V. Snegirev, Yu.M. Zinoviev, On gravitational interactions for massive higher spins in AdS_3, J. Phys. A 46 (2013) 214015, arXiv:1208.0183.
[19] M.A. Vasiliev, Free massless fields of arbitrary spin in the de Sitter space and initial data for a higher spin superalgebra, Fortschr. Phys. 35 (1987) 741.
[20] V.E. Loppatin, M.A. Vasiliev, Free massless bosonic fields of arbitrary spin in d-dimensional de Sitter space, Mod. Phys. Lett. A 3 (1988) 257.
[21] M.A. Vasiliev, Free massless fermionic fields of arbitrary spin in d-dimensional de Sitter space, Nucl. Phys. B 301 (1988) 26.
[22] S. Deser, A. Waldron, Gauge invariance and phases of massive higher spins in (A)dS, Phys. Rev. Lett. 87 (2001) 031601, arXiv:hep-th/0102166.
[23] S. Deser, A. Waldron, Partial masslessness of higher spins in (A)dS, Nucl. Phys. B 607 (2001) 577, arXiv:hep-th/0103198.
[24] S. Deser, A. Waldron, Null propagation of partially massless higher spins in (A)dS and cosmological constant speculations, Phys. Lett. B 513 (2001) 137, arXiv:hep-th/0105181.